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Oscillatory and Nonoscillatory Behaviour of Third order Neutral Delay Difference Equations

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ABSTRACT

In this note we consider the nonlinear third order neutral delay difference equation
$\Delta\,(p_n\Delta^2(y_n+h_n y_{n-k}))+q_n f\,(y_{n-l})=0$--(1)
Examples are given to illustrate the results.

KEY WORDS: Difference equation, oscillation, neutral Delay.

1. INTRODUCTION

We are concerned with the oscillatory properties of all the solutions of a third order nonlinear neutral delay difference equation of the form
$\Delta(p_n\Delta^2(y_n+h_n y_{n-k}))+q_n f(y_{n-l})=0$------(1)
Where Δ is the forward difference operator defined by $\Delta y_n=y_{n+1}-y_n$, where k, l are fixed nonnegative integers and $\{p_n\}$ $\{q_n\}$ & $\{h_n\}$ are real sequences with respect to the difference equations (1) throughout we shall assume that the following conditions hold
(H_1) $\{p_n\}\}$ $\{q_n\}$ and $\{h_n\}\geq 0$ and $q_n\neq 0$ for infinitely many value of n
(H_2) f:R→R is continuous and yf(y)≥0 for all y≠0
(H_3) There exists a real valued function g such that f(u)-f(v)=g(u,v)(u-v) for all u≠0 and v≠ 0 and g(u,v)≥L≥0ϵR
(H_4) M=max {k,l} and n_0 be a fixed non negative integers
(H_5) $R_+=\{\{y_n\}\epsilon S$: there exists an integer NϵZ such that $y_n\ \Delta y_n\geq 0$ for all n≥N} and S is the set of all nontrivial solution of (1). By a solution of equation (1) we mean a real sequence $\{y_n\}$ satisfying (1) for $n\geq n_0$.
A solution of $\{y_n\}$ is said to be oscillatory if it is neither eventually negative otherwise it is called non-oscillatory.
For more details on oscillatory behaviour of difference equations one can refer (Selvaraj, 2010; Greaf, 1999; Thandapani, 1994, 1995, 2004).

2. METHODS & MATERIALS

1. Variation of parameter
2. Annihilator method
3. Oscillation and Nonoscillation

3. RESULTS

Theorem 1: With respect to the difference equation (1) assumes that the following hold:
(C_1) $\{h_n\}$ is nonnegative and non- decreasing for all nϵz
$(C_2)\lim_{n\to\infty} sup\sum_{s=n_0}^{n-1} q_{s+1}=\infty;n_0\epsilon$z
$(C_3)\sum_{s=n_0}^{\infty}\frac{1}{q_s}=\sum_{s=n_0}^{\infty}\frac{1}{h_s}=\infty$
Then $R_+=\phi$

Proof: Suppose that the equation (1) has a solution $\{y_n\}\ \epsilon\ R_+$ since $y_n\Delta y_n\geq 0$ for all n≥N implies that $\{y_n\}$ is a nonoscillatory without loss of generality we can assume that exists an integer $n_1\geq n_0$ such that
$y_n\geq 0$, $\Delta y_n\geq 0$,
$y_{n-m}\geq 0$ &$\Delta y_{n-m}\geq 0$ for all $n\geq n_1$.
Infact for all $y_n\leq 0$ $y_{n-m}\leq 0$ for all large nϵz
the proof is similar
set $z_n=y_n+h_n y_{n-k}$ then in view of
(C_1) $z_n\geq 0$ $\Delta z_n\geq 0$ and $\Delta^2 z_n\geq 0$ for all $n\geq n_1$.
Dividing equation (1) by f(x_{n-l}) and summing from n_1 to n-1 we have

$$\frac{p_n\Delta^2 z_n}{f(x_{n-l})}-\frac{p_{n_1}\Delta^2 z_{n_1}}{f(x_{n_1}-l)}+\sum_{s=n_1}^{n-1}(\frac{p_s\ \Delta^2 z_s)g(y_{s+1-l},y_{s-l})\Delta y_{s-l}}{f(y_{s+1-l})f(y_{s-l})}=-\sum_{s=n_1}^{n-1} q_{s+1}$$

and hence

$$\frac{p_n\Delta^2 z_n}{f(x_{n-l})}-\frac{p_{n_1}\Delta^2 z_{n_1}}{f(x_{n_1-l})}\leq-\sum_{s=n_1}^{n-1} q_{s+1}$$

Thus from (C_2) we find $p_n\,\Delta^2 z_n\to\infty$ as $n\to\infty$. This implies that
$P_n\,\Delta^2\, z_n<-k_1,\ k>0$

Summing this last inequality from n_2 to n – 1 we obtain

$P_n \Delta^2 z_n < - k_1, k_1 \geq 0$

Summing the last inequality from

N_2 to n – 1 we obtain

$\Delta^2 z_n < - k_1 \sum_{s=n_2}^{n-1} \frac{1}{p_s} + p_{n_2} \Delta^2 z_{n_2} \geq n_1$

Thus from (C_3) $\Delta^2 z_n \rightarrow -\infty$ as $n \rightarrow \infty$

Which is contradict is the assumption that

$\Delta^2 z_n \geq 0$ for all large n

Example: consider the difference equation

$\Delta((n+1)\Delta^2(y_n+\frac{n-1}{n}y_{n-1})+2(3n+2)f(y_n)=0$---(A)

$\sum_{n=1}^{\infty} p_n = \sum_{n=1}^{\infty}(n+1) = \infty, \sum_{n=1}^{\infty} q_{n+1} = \sum_{n=1}^{\infty} 2(3n+2) = \infty$

Hence all the assumption of theorem holds. Hence the equation (A) has a

Solution $\{y_n\} = \{1/n\} \subset R$ such $y_n \Delta y_n \leq 0$

Theorem 2: With respect to the difference eqation (1) assume that in addition to the Condition (c_3) the following holds

(c_4) $k \geq 1$ and $- 1 \leq h_n \leq 0$

(c_5) $q_n \geq 0$ for all $n \geq n_0$

(c_6) $lim_{n\rightarrow\infty} \sum_{s=n_0}^{n-1} q_{s+1} = \infty$

Then $R_+ = \varphi$

Proof: As in theorem as have a solution

$\{y_n\} \epsilon R_+$ such that $y_n > 0$

$\Delta y_n \geq 0$,

$y_{n-m} \geq 0$ at $\Delta y_{n-m} \geq 0$ for all $n \geq n_1 \geq n_0$

Again set $z_n = y_n + h_n y_{n-k}$

than in view of (c_4) and the fact that $\{y_n\} \epsilon R_+$

We have $z_n = y_{n-k} + h_n y_{n-k}$ than is view of (c_4) and the fact that $\{y_n\} \epsilon R_+$

We have $z_n = y_{n-k} + h_n y_{n-k} \geq 0$ for all $n \geq n_1$. Since equation (1) is the same as

$\Delta(p_n \Delta^2 z_n) = -q_{n+1} f(x_{n+1-l})$, $n \geq n_1$

from condition(c_5) it follows that $\{p_n \Delta^2 z_n\}$ is non increasing for all $n \geq n_1$

Now suppose that $p_n \Delta^2 z_n < 0$for all $n \geq n_1$

$p_{n\Delta}{}^2 z_n < - k_2$, $k_2 > 0$

summing the last inequality from n_3 to n – 1 we have $\Delta^2 z_n \leq - k_2 \sum_{s=n3}^{n-1} \frac{1}{p_s} + \Delta^2 z_{n2}$ $n_3 \geq n_1$

Let $n \rightarrow \infty$ and because of (c_3) to see that

$\Delta^2 z_n \rightarrow -\infty$ that $\Delta^2 z_n \leq - k_3$, $k_3 > 0$ summing the last inequality from n_4 to n -1 we obtain

$Z_n \leq - k_3 \sum_{s=n_4}^{n-1} \sum_{k=n_4}^{m-1} \frac{1}{pks} + z_{n_4}$ $n \geq n_4$

Let $n \rightarrow \infty$ and because of c_3 we see that $z_n \rightarrow \infty$ a contradicts

Thus $p_n \Delta^2 z_n \geq 0$ now following as in theorem and using the condition

(c_5) we obtain $lim_{n\rightarrow\infty} \frac{p_{n\Delta^2 z_n}}{f(x_{n-l})} = \infty$

which is the required contradiction

Example: Consider the difference equation $\frac{1}{n}\left(\frac{n^2/n+1}{3}\right)\Delta^2\left(y_{n-2y_{n-1}}\right) + \frac{1}{n^2-1}$ f (y_n) = 0, n > 1 and f(x)=x.----(B)

Here $q_n = \frac{1}{n^2-1}$ and $\sum_{n=1}^{\infty} q_{n-1} = \frac{1}{n^2-1} \leq \infty$.

For the difference equation, assumption (C_3)and(C_5)hold:

but (C_4) and (C_6) are violated. Hence the equation (B) has a solution

$\{y_n\} = \{\frac{n-1}{n}\} \epsilon R_+$, since $y_n \Delta y_n \geq 0$

4. CONCLUSION

Based on Theorems 1, the equation (1) is oscillatory and by theorems 2 the equation (1) is non Oscillatory.

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